Low magnetic field anomaly of the Hall effect in disordered two-dimensional systems: Interplay between weak localization and electron-electron interaction

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The nonlinear behavior of the Hall resistivity at low magnetic fields in single quantum well GaAs/In$_x$Ga$_{1-x}$As/GaAs heterostructures with degenerated electron gas is studied. It has been found that this anomaly is accompanied by the weaker temperature dependence of the conductivity as compared with that predicted by the first-order theory of the quantum corrections to the conductivity. We show that both effects in strongly disordered systems stem from the second order quantum correction caused by the effect of weak localization on the interaction correction and vice versa. This correction contributes mainly to the diagonal component of the conductivity tensor, it depends on the magnetic field such as the weak localization correction and on the temperature like the interaction contribution.

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I. INTRODUCTION

The quantum corrections to the conductivity, namely the interference or weak localization (WL) correction and correction due to electron-electron (e-e) interaction, wholly determine the temperature and magnetic field dependences of the conductivity (σ) at $T \ll E_F, \tau^{-1}$, where E_F and τ are the Fermi energy and the transport relaxation time, respectively (hereafter we set $k_B = \hbar = 1$ for brevity).[1] The modern theory being elaborated since 1980[2–6] allows ones to describe most of experimental results obtained on the well controlled semiconductor two-dimensional (2D)systems quantitatively. However, one peculiarity, namely the low magnetic field dependence of the Hall coefficient (R_H), referred as beak in what follows, remains a puzzle. The magnetic field scale of the beak is close to the transport magnetic field $B_{tr} = \hbar/2el^2$, where l is the mean free path, i.e., close to the field, in which the main part of the interference correction is suppressed. As a rule, the Hall coefficient increases in absolute value with the growing magnetic field, and the magnitude of the beak is close to that of the positive magnetoconductivity caused by suppression of the weak localization: $|\delta R_H/R_H| \sim \delta\sigma^{WL}/\sigma$. The existence of low-field anomaly in R_H was pointed out in the pioneering papers on the quantum corrections.[7–9] In the later papers the anomaly of the R_H behavior is not mentioned, although the beak is observed practically in all the 2D structures.[10–12]

Theories of the weak localization and interaction corrections do not predict any low magnetic field dependence of the Hall coefficient. The WL theory asserts that the quantum interference renormalizes the transport relaxation time and, consequently, does not lead to correction in the Hall coefficient. The e-e interaction within the diffusion regime, $T\tau \ll 1$, contributes to the longitudinal conductivity σ_{xx} only and this correction does not depend on the magnetic field when the Zeeman splitting is less than the temperature, $|g|\mu_B B < T$. So, this correction leads to the temperature dependence of the Hall coefficient, while in the magnetic field R_H remains constant.[13] In Ref. 14, the sharp magnetic field dependence of the Hall coefficient in 2D systems with strong scatterers is predicted due to classical memory effects. However, manifestations of the memory effect should be independent of temperature in dirty systems where the ionized impurities rather than phonons control the scattering at low temperatures. Moreover, the Hall coefficient should decrease in magnitude with the growing magnetic field according to Ref. 14. Most experimental observations including that reported here disagree with these predictions. Thus, the origin of the beak in the B dependence of the Hall coefficient remains enigmatic.

We have analyzed numerous experimental data regarding the low-field anomaly of the Hall coefficient for more than thirty GaAs/In$_x$Ga$_{1-x}$As/GaAs and Al$_x$Ga$_{1-x}$As/GaAs/Al$_x$Ga$_{1-x}$As structures both with the electron and hole 2D gas with the carrier density from 1×10^{11} to 2×10^{12} cm^{-2} and the mobility from 1×10^2 to 2×10^4 cm^2/Vs. We have not found any correlation between the beak magnitude and such the structure parameters as the transport and quantum relaxation time, carriers density, spin-orbit interaction strength and so on. In our opinion it indicates that there is no universal reason for such a behavior of the Hall coefficient. In this paper, we show that in strongly disordered structures in deep diffusion regime all the transport properties, including the magnetic and temperature dependence of R_H, are described with taking into account the interplay between the weak localization and interaction effects. This interplay term contributes to σ_{xx} only like the interaction correction and depends on the magnetic field like the WL correction.

II. EXPERIMENTAL DETAILS

The structures investigated were grown by metal-organic vapor-phase epitaxy on a semiinsulating GaAs substrate and consist of 0.5-μm-thick undoped GaAs epilayer, a In$_x$Ga$_{1-x}$As quantum well with Sn or Si δ layer situated in the well center and a 200 nm cap layer of undoped GaAs. The samples were mesa etched into standard Hall bars and

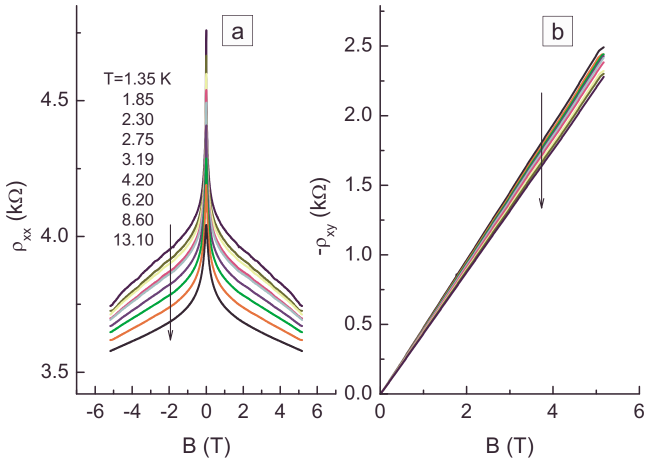

FIG. 1. (Color online) The magnetic field dependences of ρ_{xx} (a) and ρ_{xy} (b) for V_g=−1 V taken at different temperatures. The arrows indicate the temperature growth.

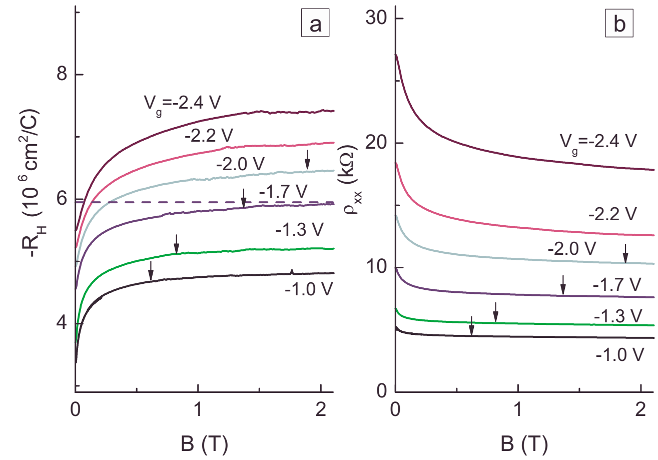

FIG. 2. (Color online) The magnetic field dependences of R_H (a) and ρ_{xx} (b) taken for different gate voltages at T=1.4 K. The dashed line is the Hall constant for V_g=−1.7 V obtained from the linear interpolation of ρ_{xy} within the low magnetic field domain, $|B|\lesssim 5B_{tr}$. The arrows indicate the B_{tr} values.

then an Al gate electrode was deposited by thermal evaporation onto the cap layer through a mask. Varying the gate voltage (V_g) we were able to change the electron density (n) and the conductivity of 2D electron gas in the quantum well. We investigated samples prepared from four wafers with different well width, doping level and well composition. All the measurements were carried out in the Ohmic regime using DC technique. The results obtained were mostly analogous, therefore we will discuss the results for the structure 4261 studied more thoroughly. The quantum well width in this structure is 8 nm, indium content in the quantum well is 0.2 and tin density in δ layer is about 2×10^{12} cm^{-2}.

III. RESULTS AND DISCUSSION

The magnetic field dependences of ρ_{xx} and ρ_{xy} for V_g=−1 V taken at different temperatures are shown in Figs. 1(a) and 1(b), respectively. They are typical for such a type of systems. The sharp negative magnetoresistance at low magnetic field [Fig. 1(a)] results from suppression of the WL contribution. A crossover to the paraboliclike behavior of ρ_{xx} at $B\gtrsim 2$ T and the decrease in ρ_{xy} with the temperature increase [see Fig. 1(b)] come from the e-e interaction correction. At first sight, ρ_{xy} linearly depends on the magnetic field [Fig. 1(b)] as predicted theoretically. Let us, however, inspect the Hall coefficient, $R_H=\rho_{xy}/B$, which magnetic field dependences taken for different gate voltages at T=1.4 K are plotted in Fig. 2(a). It is evident that R_H decreases in magnitude when B goes to zero for all the gate voltages. Comparing these dependences with that for magnetoresistance [presented in Fig. 2(b)], one can see that the characteristic scales in B domain for the R_H beak and for the interference induced negative magnetoresistance are close; the main changes happen at $B\lesssim B_{tr}$ in both cases. Therefore, before discussing the low-field peculiarity of the Hall coefficient, we would like to analyze the contributions of the interference and interaction.

First we remind the reader of the basic results of the quantum correction theory that will be used for analysis. We will restrict ourselves to the WL and e-e interaction corrections and neglect the corrections in the Cooper channel (for more details, see discussion at the end of this section). The expression for the conductivity tensor components taking into account the first order in $\delta\sigma/\sigma$ corrections are the following:

$$\sigma_{xx}(B,T)=\frac{en\mu(B,T)}{1+[\mu(B,T)B]^2}+\delta\sigma_{xx}^{ee}(T), \quad (1)$$

$$\sigma_{xy}(B,T)=\frac{en\mu(B,T)^2B}{1+[\mu(B,T)B]^2}. \quad (2)$$

In the actual case of $T\tau\ll 1$, the correction $\delta\sigma_{xx}^{ee}$ is just the Altshuler-Aronov (AA) correction given by[2,15–19]

$$\delta\sigma^{\mathrm{AA}}(T)=K_{ee}^{\mathrm{AA}}G_0\ln(T\tau), \quad (3)$$

where

$$K_{ee}^{\mathrm{AA}}=1+3\left[1-\frac{1+\gamma_2}{\gamma_2}\ln(1+\gamma_2)\right] \quad (4)$$

with $G_0=e^2/\pi h$ and γ_2 standing for the Landau's Fermi liquid amplitude. Because the WL correction is reduced to the renormalization of the transport relaxation time,[20] it is incorporated in Eqs. (1) and (2) into the mobility μ in such a way that

$$\delta\sigma^{WL}(B,T)=en\,\delta\mu(B,T), \quad (5)$$

where

$$\delta\sigma^{WL}(0,T)=G_0\ln\left[\frac{\tau}{\tau_\phi(T)}\right], \quad (6)$$

and $\Delta\sigma^{WL}(B,T)=\delta\sigma^{WL}(B,T)-\delta\sigma^{WL}(0,T)$ is described by the expression[21,22]

$$\Delta\sigma^{WL}(B,T)=\alpha G_0\mathcal{H}\left[\frac{\tau}{\tau_\phi(T)},\frac{B}{B_{tr}}\right],$$

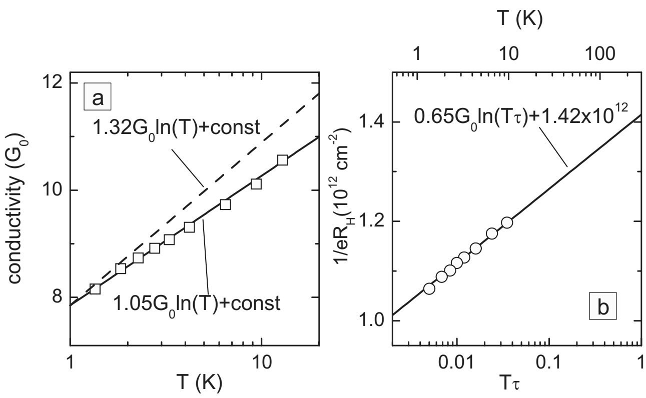

FIG. 3. The temperature dependences of σ at $B=0$ (a) and $[eR_H(B=4\ \text{T})]^{-1}$ (b) for $V_g=-1.7$ V ($\tau=2.7\times10^{-14}$ s).

$$\mathcal{H}(x,y)=\psi\left(\frac{1}{2}+\frac{x}{y}\right)-\psi\left(\frac{1}{2}+\frac{1}{y}\right)-\ln x. \qquad (7)$$

Here, τ_ϕ is the phase relaxation time, $\psi(x)$ is a digamma function, and α is the prefactor, whose value depends on the conductivity if one takes into account two-loop localization correction and the interplay of the weak localization and interaction,[23]

$$\alpha\simeq1-\frac{2G_0}{\sigma},\quad \sigma<2G_0 \qquad (8)$$

We turn now to the analysis of the data. By way of example we consider the case of $V_g=-1.7$ V. As seen from Fig. 3(a) the temperature dependence of the conductivity without magnetic field is close to the logarithmic one, $\sigma(T)=\beta\ln(T/T_0)$, with the slope β equal to 1.05 ± 0.05. To find what portion of the slope comes from WL let us inspect the low-field magnetoconductivity [Fig. 4(a)]. The electron density $n=(1.42\pm0.03)\times10^{12}$ cm^{-2} needed for the analysis we obtain from the extrapolation of the temperature dependence of the Hall density $n=1/eR_H$ taken at high magnetic field, $B=4$ T, to $T\tau=1$ [Fig. 3(b)]. Such a dependence of R_H comes from the diffusion contribution of the interaction,

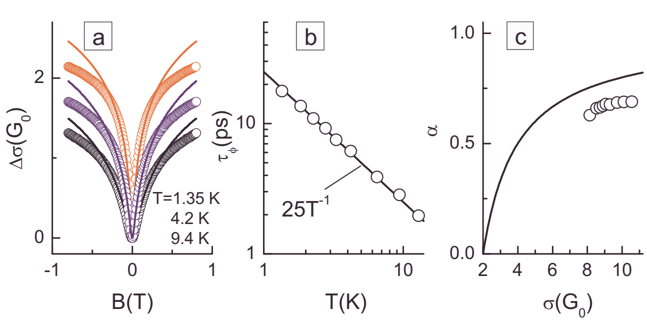

FIG. 4. (Color online) (a) The magnetic field dependence of $\Delta\sigma$ for different temperature, $V_g=-1.7$ V. Symbols are the experimental data, lines are the results of the best fit by Eq. (7) within the range of B from $-0.2B_{tr}$ to $0.2B_{tr}$, where $B_{tr}=1.26$ T. (b) The temperature dependences of τ_ϕ found from the fit. (c) The prefactor α plotted as a function of the conductivity at $B=0$ driven by the temperature. The solid line is Eq. (8).

which vanishes at $T\tau=1$. So, the value of $1/eR_H$ at $T\tau=1$ actually gives the electron density. An analysis shows that Eq. (7) describes the experimental dependences $\Delta\sigma(B)=1/\rho_{xx}(B)-1/\rho_{xx}(0)$ well [see Fig. 4(a)]. The values of τ_ϕ and α found from the fit within magnetic field range $|B|<0.2B_{tr}$ for the different temperatures are plotted in Figs. 4(b) and 4(c), respectively. One can see that $\tau_\phi(T)$ is very close to $1/T$. The prefactor values being noticeably less than unity, $\alpha=0.6\ldots0.7$, decreases slightly with the decreasing conductivity. As Fig. 4(c) shows, such a behavior agrees well with the theoretical result, Eq. (8). So, the value of τ_ϕ found from the fit of Eq. (7) to the data is the value of the phase breaking time. As mentioned above it is inversely proportional to the temperature in whole agreement with theoretical prediction.[2] Thus, taking into account Eq. (6) we conclude that the weak localization gives the unit in the slope of the σ vs $\ln T$ dependence at $B=0$.

Now let us determine the interaction contribution to the conductivity. It can be found from the temperature dependence of the Hall coefficient at high magnetic field [see Fig. 3(b)] because $\delta R_H/R_H\simeq-2\delta\sigma_{xx}^{ee}/\sigma_0$ under the condition $|\delta\sigma_{xx}^{ee}|\ll\sigma_0$. This gives $\delta\sigma_{xx}^{ee}\simeq0.32\ln T\tau$. However, the more straightforward way (which does not require the fulfilment of this condition) is the following.[24] Since the interaction in the diffusion regime contributes to σ_{xx} only, one should find such the contribution to the conductivity which exists in σ_{xx} but is absent in σ_{xy}. It can be done by expressing $\mu(B,T)$ from Eq. (2) and substituting it in Eq. (1). Doing so, we obtain the expression

$$\delta\sigma_{xx}^{ee}=\frac{1}{\rho_{xx}^2+\rho_{xy}^2}\left[\rho_{xx}-\rho_{xy}\sqrt{\frac{en(\rho_{xx}^2+\rho_{xy}^2)}{\rho_{xy}B}-1}\right] \qquad (9)$$

that allows us to find $\delta\sigma_{xx}^{ee}$ using the experimental quantities ρ_{xx} and ρ_{xy}. The magnetic field dependences of $\delta\sigma_{xx}^{ee}$ found by this way at different temperatures are presented in Fig. 5(a). One can see that $\delta\sigma_{xx}^{ee}$ is practically independent of the magnetic field while $B\gtrsim(1.5-2)$ T. The temperature dependence of $\delta\sigma_{xx}^{ee}$ is logarithmic, and the slope K_{ee}^{exp} being equal to 0.32 ± 0.05 remains independent of the magnetic field at $B\gtrsim2$ T [see Figs. 5(b) and 5(c)]. Such the behavior agrees well with Eq. (3) and, thus, $K_{ee}^{\text{exp}}=0.32\pm0.05$ is just the value of K_{ee}^{AA}. Using Eq. (4) one obtains $\gamma_2\simeq0.53$ that is in a good agreement with the results of Ref. 4 if one takes into account the renormalization effect.[24,25] Thus, the T dependence of $\delta\sigma_{xx}^{ee}$ in the high magnetic field, $B\gtrsim(1.5-2)$ T, is determined by the AA quantum correction.

As we have obtained the value of K_{ee}^{AA} responsible for the AA contribution, we can compare the value of β describing the experimental T dependence of σ at $B=0$ with the value of $1+K_{ee}^{\text{AA}}$ (recall that 1 comes from the WL effect) found from the analysis of the data in the magnetic field. If the model used and, consequently, Eqs. (1)–(8), are correct, the values of $1+K_{ee}^{\text{AA}}$ and β should be equal to each other. We have experimentally that $\beta=1.05\pm0.05$ [see Fig. 3(a)] is visibly less than $1+K_{ee}^{\text{AA}}=1.32\pm0.05$. The reason for this discordance is transparent. The K_{ee}^{AA} value has been above obtained at relatively strong magnetic field, where the interaction contribution does not depend on the magnetic field. However,

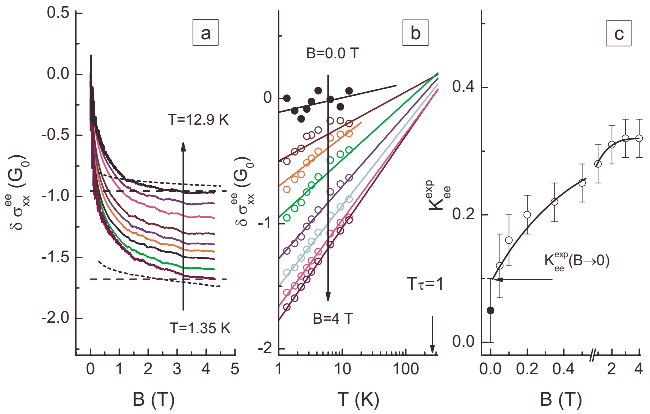

FIG. 5. (Color online) (a) The magnetic field dependences of the $\delta\sigma_{xx}^{ee}$ for different temperatures: $T=1.35$, 1.8, 2.27, 2.77, 3.3, 4.2, 6.5, 9.4, and 12.9 K. The dashed lines are $K_{ee}^{AA}\ln T\tau$ for $T=1.35$ K and 12.9 K with $\tau=2.7\times 10^{-14}$ s and $K_{ee}^{AA}=0.32$ found from the T dependence of R_H in high magnetic field [see Fig. 3(b)]. The dotted lines are $\delta\sigma_{xx}^{ee}$ obtained for $T=1.35$ K and 12.9 K in the assumption that R_H is independent of B. (b) The temperature dependence of $\delta\sigma_{xx}^{ee}$ for different magnetic field: $B=0$, 0.05, 0.1, 0.2, 0.5, 1.0, 2.0, and 4.0 T. Data at $B=0$ are obtained by extrapolation of $\delta\sigma_{xx}^{ee}$ vs B curves shown in the panel (a) to $B=0$ as described on page 5. (c) The B-dependence of the slope K_{ee}^{exp} of the $\delta\sigma_{xx}^{ee}$ vs $\ln T$ dependences shown in the panel (b).

close inspection of Fig. 5 shows that not only $\delta\sigma_{xx}^{ee}$ diminishes in absolute value at $B\to 0$ but the slope K_{ee}^{exp} decreases as well. Of course, the accuracy of K_{ee}^{exp} determination is not very high in low magnetic field. As seen from Fig. 5(a) the experimental $\delta\sigma_{xx}^{ee}$ vs B plots are very noisy near $B=0$. This is because the expression, Eq. (9), used for the data treatment contains B in the denominator. Nevertheless, extrapolating the K_{ee}^{exp} vs B data to $B=0$ one obtains $K_{ee}^{exp}(B\to 0)=0.1\pm 0.05$ [shown by arrow in Fig. 5(c)]. Together with 1 coming from the WL effect we have the value 1.1 ± 0.05 nearly equal to $\beta=1.05\pm 0.05$.

In principle, the K_{ee}^{exp} change could be induced by the K_{ee}^{AA} decrease caused by suppression of two of three triplet channels in Eq. (3) due to the Zeeman effect.[17,19,26–28] However, this effect is negligible in our case for the low value of the effective g-factor, $g\sim 0.5$. Moreover, if the Zeeman splitting would be important, the T dependence of $\delta\sigma_{xx}^{ee}$ in high magnetic field should be strongly nonlogarithmic as it takes place in 2D hole gas (see Fig. 2 in Ref. 28).

It is essential to note that the strong decrease in $\delta\sigma_{xx}^{ee}$ in absolute value with the lowering magnetic filed and the beak in R_H vs B dependence are closely related. Really, if one uses the linear interpolation of ρ_{xy} within the range $\pm(4-5)B_{tr}$ in the above procedure, i.e., one supposes that R_H is constant as shown in Fig. 2(b) for $V_g=-1.7$ V by the dashed line, we obtain $\delta\sigma_{xx}^{ee}$, which is practically independent of the magnetic field [dotted lines in Fig. 5(a)].

Thus, there is common reason behind the beak in the R_H vs B dependence and the existence of magnetic field dependence of $\delta\sigma_{xx}^{ee}$. Because $|\delta\sigma^{WL}|$ is about σ at low temperatures, it is natural to assume that the second order corrections play an important role under our conditions.

The second order effects are studied in the number of papers.[3,23,29–34] The second-order interaction correction (not involving Cooperons), $\delta\sigma_2^{\mathrm{I}}$, logarithmically depends on the temperature, but does not depend on the magnetic field analogously to the AA correction.[33,34] That is why it barely gives the correction to K_{ee}^{AA}, Eq. (4), and does not affect the low magnetic field magnetoresistance $\Delta\sigma(B)$. The other two second order terms have an impact on $\Delta\sigma(B)$. They are $\delta\sigma^{\mathrm{WL}\times\mathrm{I}}$ coming from the interplay between the weak localization and the interaction effects,[3] and $\delta\sigma_2^{\mathrm{WL}}$, which is the second order interference correction.[23] Except for opposite sign, the magnetic field dependences of both terms are close to that for the first-order interference correction. Namely this fact results in the appearance of $\alpha<1$ in Eq. (7). Since the interference correction stems from the (B dependent) correction to the impurity scattering cross section and hence renormalizes the value of the elastic transport scattering rate $1/\tau$, the higher order interference corrections do not contribute to the Hall effect[35,36] analogously to the first order one.[2] Moreover, $\delta\sigma_2^{\mathrm{WL}}$ does not contribute to the T dependence of σ at zero magnetic field, since the terms of the second and third orders cancel out in the interference correction at $B=0$.[29,30] Thus, it is reasonable to assume that the main effect comes from the interplay term $\delta\sigma^{\mathrm{WL}\times\mathrm{I}}$.

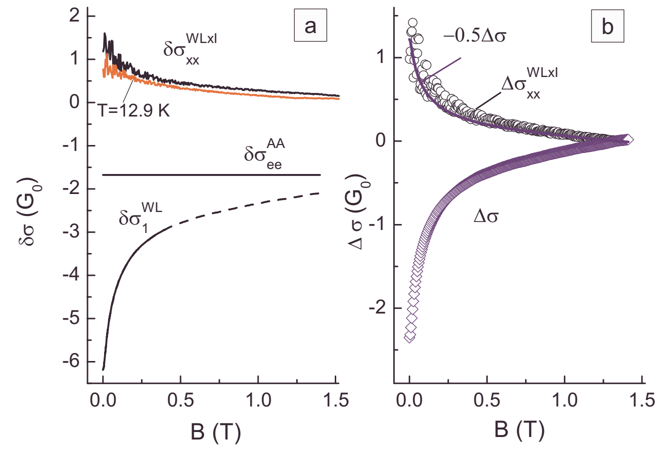

FIG. 6. (Color online) (a) The magnetic field dependences of $\delta\sigma_1^{\mathrm{WL}}$, $\delta\sigma_{ee}^{\mathrm{AA}}$ and $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$ for $T=1.35$ K. The dashed line indicates that the estimate for $\delta\sigma_1^{\mathrm{WL}}$ is rough in this range, because Eq. (10) is valid when $B\ll B_{tr}$. (b) The experimental dependences $\Delta\sigma(B)=\rho_{xx}^{-1}(B)-\rho_{xx}^{-1}(B_{tr})$ and $\Delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}(B)=\delta\sigma_{xx}^{\mathrm{WL}\times I}(B)-\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}(B_{tr})$.

Generally, the interplay effect may give corrections to both components of the conductivity tensor. We designate them as $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$ and $\delta\sigma_{xy}^{\mathrm{WL}\times\mathrm{I}}$. Because $\delta\sigma_{xy}^{\mathrm{WL}\times\mathrm{I}}=0$ at $B=0$, the difference between β and $1+K_{ee}^{\mathrm{AA}}$ results from $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$. Suppose that $\delta\sigma_{xy}^{\mathrm{WL}\times\mathrm{I}}$ is small in the presence of magnetic field as well: $\delta\sigma_{xy}^{\mathrm{WL}\times\mathrm{I}}\ll\mu B\,\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$. In this case the quantity $\delta\sigma_{xx}^{ee}$ found above is just the sum of the AA correction, which is independent of the magnetic field and logarithmically dependent on the temperature, and the second order correction $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$, which depends both on T and B. In Fig. 6(a), the value of $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$ found as $\delta\sigma_{xx}^{ee}-K_{ee}^{\mathrm{AA}}\ln T\tau$ with $\tau=2.7\times 10^{-14}$ s and $K_{ee}^{\mathrm{AA}}=0.32$ are plotted against the magnetic field. For comparison, the first-order corrections $\delta\sigma_{ee}^{\mathrm{AA}}$ and $\delta\sigma_1^{\mathrm{WL}}$ are depicted in the same figure. The $\delta\sigma_1^{\mathrm{WL}}$ correction is obtained from the experimental data in accordance with Eqs. (6) and (7) as follows

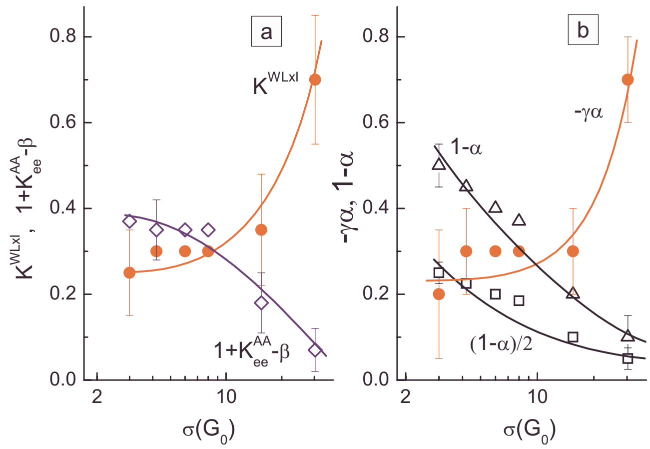

FIG. 7. (Color online) (a) The values of $K^{\mathrm{WL}\times\mathrm{I}}$ (circles) and the difference between $1+K_{ee}^{\mathrm{AA}}$ and β (diamonds) as a function of the conductivity at $T=1.35$ K. (b) The conductivity dependence of $-\gamma\alpha$ (circles), $1-\alpha$ (triangles), and $(1-\alpha)/2$ (squares). Lines are provided as a guide to the eye. The conductivity in both panels is driven by the gate voltage.

$$\delta\sigma_1^{\mathrm{WL}} \simeq \frac{\Delta\sigma(B)}{\alpha} - \ln\left(\frac{\tau_\phi}{\tau}\right). \tag{10}$$

Two important properties of $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$ are evident from the figure. First, the interplay correction is metalliclike in contrast to the WL and AA corrections, i.e., it increases with the temperature decrease [see Fig. 6(a)]. Qualitatively this explains the difference between $1+K_{ee}^{\mathrm{AA}}$ and β. Second, the B range, where the main changes in $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$ occur, is the same as for $\delta\sigma_1^{\mathrm{WL}}$: $B<B_{tr}\simeq1.36$ T. The fact that $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$ vs B curve is close in the shape to the low magnetic field magnetoconductance is illustrated by Fig. 6(b). As seen $\Delta\sigma(B)$ multiplied by the factor γ of -0.5 fits the $\Delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$ dots rather well. It is vital to note that both properties are in agreement with the theoretical prediction.[3]

The second property has been used to obtain the value of $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}(B=0)$ [and $\delta\sigma_{xx}^{ee}(B=0)$ shown in Fig. 5(b) by solid circles]. We have interpolated the experimental B dependences of $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$ (and $\delta\sigma_{xx}^{ee}$) at $B<B_{tr}$ by the experimental curve $\Delta\sigma(B)$ excluding the noisy vicinity of $B=0$ and extrapolated it to $B=0$. The T dependence of $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}(0)$ found in such a way is close to the logarithmic one with the slope $K^{\mathrm{WL}\times\mathrm{I}}=-0.3\pm0.1$.

Thus, the temperature dependence of the conductivity at $B=0$ caused by all three contributions $\delta\sigma_1^{\mathrm{WL}}$, $\delta\sigma_{ee}^{\mathrm{AA}}$, and $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$ is very close to that observed experimentally. The total slope equal to $1+K_{ee}^{\mathrm{AA}}+K^{\mathrm{WL}\times\mathrm{I}}=1.02\pm0.1$ is in a good agreement with the experimental value $\beta=1.05\pm0.05$ [see Fig. 3(a)].

Analysis described has been performed within wide range of the conductivity driven by the gate voltage. The values of $K^{\mathrm{WL}\times\mathrm{I}}$ plotted against σ at $T=1.35$ K are shown in Fig. 7(a). In the same figure the difference between $1+K_{ee}^{\mathrm{AA}}$ and β is depicted. It is seen that both data are close to each other at low conductivity, $\sigma<20G_0$. At higher conductivity, they diverge drastically.

The contribution of $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$ to the magnetoconductivity is illustrated by Fig. 7(b). We characterize it by the product $\gamma\alpha$ which is $\Delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}(B)$ to $\Delta\sigma_1^{\mathrm{WL}}(B)$ ratio. If one supposes that $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$ is alone and there is no $\Delta\sigma_2^{\mathrm{WL}}$ contribution to the magnetoconductivity, this value should be equal to $1-\alpha$. If these corrections are the same in magnitude (as it turns out theoretically for the short-range interaction[23]), the $\gamma\alpha$ value has to be equal to half of this value, $(1-\alpha)/2$. As seen from Fig. 7(b) the agreement is satisfactory with both the cases at $\sigma<(10-15)G_0$ if one takes into account the experimental error. At $\sigma\simeq30G_0$ the difference becomes crucial.

The discrepancy between the data obtained in different manner evident in Figs. 7(a) and 7(b) at high conductivity probably means that our assumption about smallness of the correction to the Hall conductivity σ_{xy} is not valid in this case suggesting further investigations are needed to understand the origin of the low-field anomaly in the Hall effect in the relatively clean systems.

Thus, the second-order correction $\delta\sigma_{xx}^{\mathrm{WL}\times\mathrm{I}}$ caused by the interplay between the WL and interaction corrections is of importance in our case. At low conductivity, $\sigma<(10-15)G_0$, this correction contributes to the diagonal component of the conductivity tensor σ_{xx} only. Its temperature dependence is metalliclike in contrast to the WL and AA corrections, which are insulating. Its magnetic field dependence is close in the shape to that of the WL correction, although the magnetoconductivity itself is negative in contrast to that induced by suppression of the weak localization. Existence of this correction results in: (i) the depressing of the interference induced low magnetic field magnetoresistance; (ii) the difference between the slope of the σ vs $\ln T$ dependence and the value of $1+K_{ee}^{\mathrm{AA}}$; (iii) the occurrence of the beak in the R_H vs B dependence in low magnetic field.

Before concluding the paper, let us discuss the interaction corrections in the Cooper channel, which has been neglected in the above analysis. Two terms are distinguished in the Cooper channel. They are the Maki-Thomson correction and the correction to the density of states (DoS). The role of these terms in the low field magnetoconductivity is thoroughly considered in Ref. 23. Concerning the Hall effect, only the DoS term can influence it.[2] This is because the DoS correction contributes to σ_{xx} and do not to σ_{xy} as well as the AA correction, but in contrast to it the DoS term yields the magnetoconductivity close in the shape to that of interference induced magnetoconductivity. Following the paper, Ref. 23, we have estimated the DoS correction value $\delta\sigma^{\mathrm{DoS}}$ and its contribution to the Hall coefficient $\delta R_H/R_H\simeq-2\delta\sigma^{\mathrm{DoS}}/\sigma$. It has been obtained, that $\delta\sigma^{\mathrm{DoS}}$ is practically independent of the sample parameters within actual conductivity range, $\sigma=(3-30)G_0$, and may result in about 5% depth of the beak in the Hall effect instead of (25–35)% observed experimentally [see Fig. 2(a)]. Moreover, the temperature dependence of the DoS correction at $B=0$ is too weak to resolve discrepancy between the T dependence of σ observed experimentally and predicted by the first order theory. So, we believe that the interplay effects rather than the DoS interaction correction play the key role in the appearance of the low magnetic field anomaly in the Hall effect in strongly disordered 2D systems.

IV. CONCLUSION

We have studied the nonlinear behavior of the Hall resistivity in the vicinity of zero magnetic field. Investigating the two-dimensional electron gas in strongly disordered GaAs/$In_xGa_{1-x}As$/GaAs single quantum well structures we have shown that the anomaly of the Hall resistance and impossibility of description of the temperature dependences of zero-field conductivity by taking into account only two first order WL and AA quantum corrections are explained by significant contribution of the second order correction resulting from the effect of weak localization on the interaction correction and vice versa in disordered systems with $\sigma < (10-15)G_0$. The experimental results are satisfactorily interpreted under assumption that this correction contributes to the diagonal component of the conductivity tensor only. It has been found that the magnetic field dependence of the interplay correction is close to that of the weak localization one, while its temperature dependence is metalliclike.

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